STANDARD DEVIATION

Apply this easy-to-use statistical tool when designing mixes.

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Suppose a nature guide assures you that the river ahead averages 3 feet in depth. Would you cross it with this information alone? Probably not. You would want to know something about the variation in depth, especially if you don't know how to swim. You might cross the river if you knew the maximum depth is 3 1/4 feet and the minimum is 2 3/4 feet. But what if the depth ranged from a half a foot to 5 1/2 feet? Both ranges average 3 feet, but they deviate differently from that average.

So it is with concrete strength. A standard deviation measures the variance of a set of numbers from the average. Standard deviations can provide the precast concrete producer with a means of evaluating mix designs for consistency, and this article explains how. But that is just the core of the matter – there are many other reasons for using standard deviations, including the fact that their use is typically mandated.

Why use standard deviations?

Standard deviations are good tools for regularly evaluating your mix designs. They provide a measurement by which to evaluate the consistency of the strengths you attain, much the same way they can help evaluate the "consistency" of the depth of the river the nature guide wants you to cross. While you can't change the profile of a river, you can tweak your production practices to get more consistent strength results.

Standard deviations are a broad measure of quality in your production operation and will provide a clear sign if changes are necessary. By continually monitoring your 28-day strength test results and evaluating the standard deviations of the strengths achieved with each mix, you can determine whether to redesign a mix or whether opportunities exist for reducing costs or improving strengths. Cost savings may result from reducing the amount of cement or by reducing or eliminating product rejections caused by low strengths. Both are dependent on the degree of control you have over your mixes.

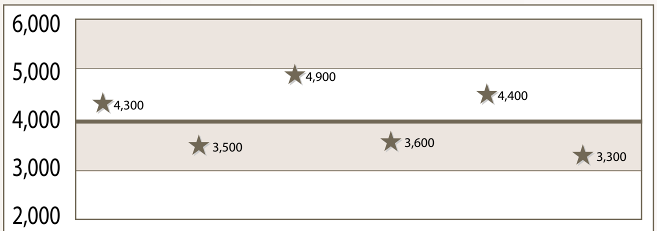
There will always be *some* variation in the strengths you achieve, so in order to consistently reach a *specified* 28-day compressive strength, known as f'_c ("f prime c"), you should work with a mix designed to deliver a higher concrete strength. This higher strength is called the *required* average compressive strength, or f'_{cr} ("f prime c required"). You must choose an f'_{cr} that will ensure all your products achieve the specified strength or greater – obviously you cannot do this without knowing exactly how much your strengths vary. The standard deviation is needed to determine f'_{cr}.

In addition, American Concrete Institute Building Code Requirements for Structural Concrete (ACI Code 318-02, section 5.3.1.1) mandates these measurements: "Where a concrete production facility has test records, a standard deviation shall be established."

Formulas and Definitions

Normally, you should have at least 30 consecutive tests or two groups of consecutive tests totaling 30 tests or more. With this number of tests, a result is the average of any two cylinder breaks from the same sample. Later on, we will discuss how to proceed if only 15 to 29 consecutive tests are available, or if no data is available to establish a standard deviation.

The following figure illustrates several fundamental statistical concepts.



SAMPLINGS OF 28-DAY STRENGTHS

In this abbreviated example, six 28-day strength test results have been graphed for one particular mix. The heavy horizontal line is the average strength of all six tests. In statistical language, this is identified as $\bar{X}$ (mean or average). Figure the average by adding all the test values and dividing by the number of test results, **N**:

$\bar{X}$ = (4,300 + 3,500 + 4,900 + 3,600 + 4,400 + 3,300) ÷ 6 = 4,000 psi

But if you have a 4,000 psi specification requirement, three tests did not meet the required strength. These might represent product cast on three different days or even at different times during the same day. It is useful to have a single value represent the spread of these numbers above and below the average. If we take the difference between each individual test and the average, without regard to whether they are above or below average, then add them up and divide by the total number of differences (N), the result is the *mean deviation.*

(300 + 500 + 900 + 400 + 400 + 700) ÷ 6 = 533 psi

The *mean deviation* is one measure of variability. The smaller the result – and the smaller the variance of each test from the average, $\bar{X}$ – the better your group of numbers. But you should also know that one or two large differences in a group of tests will have a significant impact on the *mean deviation.* Those large differences are disturbing because they indicate that some individual tests are not really indicative of the whole group. Perhaps someone mishandled a cylinder and left it to dry in the sun all day. Obviously, the strength result of that cylinder would stand out – it would be an "outlier."

A *standard deviation* seeks to emphasize the impact of very low or very high test values, or outliers. To do this, we apply a simple mathematical concept: we square the differences before we add them together. Then we divide by (N-1), one less than the total number of samples. *The square root of this value is known as the standard deviation.* This measure of variability, the standard deviation, is normally written as the letter s. The mathematical formula for standard deviation is:

$$s = \sqrt{\frac{\Sigma (Xi - \bar{X})^2}{(N-1)}}$$

where s = standard deviation in psi
Σ = summation of
N = number of tests
Xi = each individual test in psi
$\bar{X}$ = average strength in psi

Expanding this formula, the standard deviation of the tests given in the previous example would be:

$$s = \sqrt{\frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + (X_3 - \bar{X})^2 + (X_4 - \bar{X})^2 + (X_5 - \bar{X})^2 + (X_6 - \bar{X})^2}{(6-1)}}$$

The equations in this article can be calculated with a standard hand-held business or scientific calculator, and the answers will magically appear. But for the sake of explanation, here are the calculations by hand:

i	Xi	Xi - $\bar{X}$	(Xi - $\bar{X}$)^2
1	4,300	4,300 - 4,000 = 300	90,000
2	3,500	3,500 - 4,000 = -500	250,000
3	4,900	4,900 - 4,000 = 900	810,000
4	3,600	3,600 - 4,000 = -400	160,000
5	4,400	4,400 - 4,000 = 400	160,000
6	3,300	3,300 - 4,000 = -700	490,000
Σ	24,000		1,960,000

$$s = \sqrt{\frac{1{,}960{,}000}{5}} = 626 \text{ psi}$$

Is this considered good, bad, or what? Here are some threshold values published by ACI:

3,000 to 4,000 psi concrete		> 10,000 psi concrete
Standard Deviation	**Quality Control**	**Standard Deviation**
300 to 400 psi	Excellent	300 to 500 psi
400 to 500 psi	Good	500 to 700 psi
500 to 600 psi	Fair	
> 600 psi	Poor	> 700 psi

Besides being an indicator of your quality control, the standard deviation can determine f'_{cr} (the required average strength). As required by ACI 318-02, two formulas are used for f'_c less than or equal to 5,000 psi when at least 30 tests are available. You must choose the larger f'_{cr} from the following calculations:

$f'_{cr} = f'_c + 1.34\,s$
$f'_{cr} = f'_c + 2.33\,s - 500$

Continue with the example above, where the standard deviation was calculated as *626* psi. If you are looking for a specified 28-day strength (f'_c) of 4,000 psi, your required strength (f'_{cr}) should be the larger result of the two formulas.

$f'_{cr} = 4{,}000 + (1.34 \times 626) = 4{,}839$ psi
or
$f'_{cr} = 4{,}000 + (2.33 \times 626) - 500 = 4{,}959$ psi

Thus, in this scenario you should aim at producing 4,*959* psi concrete, or more practically 5,000 psi concrete, in order to ensure that all your break tests will be at least the specified 4,000 psi.

Now let's turn to having two sets of consecutive test records which, when added up, total at least 30 tests (on concrete made with similar materials, QC procedures and operating conditions). The following formula provides the statistical average deviation (s) of the values calculated from each set of test results:

$$s = \sqrt{\frac{(n_1 - 1)(s_1)^2 + (n_2 - 1)(s_2)^2}{(n_1 + n_2 - 2)}}$$

where n_1 = number of samples in group 1,
n_2 = number of samples in group 2, and
s_1 and s_2 are calculated with the formula above for 30 tests

When the number of tests is between 15 and 30, multiply the standard deviation calculated by the formula above with the appropriate number from the following ACI 318 table:

TABLE 5.3.1.2 — MODIFICATION FACTOR FOR STANDARD DEVIATION WHEN LESS THAN 30 TESTS ARE AVAILABLE

No. of tests*	Modification factor for standard deviation†
Less than 15	Use table 5.3.2.2
15	1.16
20	1.08
25	1.03
30 or more	1.00

* Interpolate for intermediate numbers of tests.
† Modified standard deviation to be used to determine required average strength f'_{cr} from 5.3.2.1.

Finally, if you do not have sufficient test records for calculating a standard deviation, you will need to work up a mix for a new specification. Once again, ACI has a table for it. Determine the required average strength from ACI 318, Table 5.3.2.2 (right), select mix proportions that will achieve the required average strength, and make trial mixes and/or several field tests.

TABLE 5.3.2.2 — REQUIRED AVERAGE COMPRESSIVE STRENGTH WHEN DATA ARE NOT AVAILABLE TO ESTABLISH A STANDARD DEVIATION

Specified compressive strength, f'_c, psi	Required average compressive strength, f'_{cr}, psi
Less than 3000	$f_c' + 1000$
3000 to 5000	$f_c' + 1200$
Over 5000	$1.10\ f_c' + 700$

Application

The example on page 14 represents the procedure for calculating the standard deviation for a group of 30 tests on a mix with f'_c= 4,000 psi, using the formulas presented.

The data in this example is organized in the table above to evaluate the data in accordance with ACI 318-02 Section 5.6.3.3, which requires that no 28-day average fall below the specified strength (4,000 psi) by more than 500 psi, and that the average of any three consecutive strength tests never fall below the specified strength. The data is further organized to easily compute the standard deviation.

The total number of tests (N) in this example is 30. Therefore the standard deviation is calculated as follows:

$$s = \sqrt{\frac{4{,}196{,}294}{29}} = 380 \text{ psi}$$

As per the ACI chart given earlier, this standard deviation points to excellent quality control procedures. While tests 22, 28 and 30 each resulted in a cylinder test that was below the specified strength, the other test results were high enough to assume that there may have been a problem with casting or with the cylinders themselves.

	28-DAY STRENGTHS (psi)					
TEST 1	TEST CYLINDER 1	TEST CYLINDER 2	AVERAGE OF THE TWO TESTS, Xi	THREE CONSECUTIVE 28-DAY AVERAGES	$(Xi - \bar{X})$	$(Xi - \bar{X})^2$
1	4,880	4,910	4,895		85	7,225
2	4,190	4,360	4,275		-535	286,225
3	5,000	5,200	5,100	4,757	290	84,100
4	4,590	4,705	4,648	4,674	-163	26,406
5	4,990	4,980	4,985	4,911	175	30,625
6	4,900	4,910	4,905	4,846	95	9,025
7	4,880	5,060	4,970	4,953	160	25,600
8	4,990	4,930	4,960	4,945	150	22,500
9	4,700	4,700	4,700	4,877	-110	12,100
10	4,435	4,575	4,505	4,722	-305	93,025
11	5,280	5,350	5,315	4,840	505	255,025
12	4,630	4,830	4,730	4,850	-80	6,400
13	5,510	5,595	5,553	5,199	743	551,306
14	4,800	4,760	4,780	5,021	-30	900
15	5,000	5,090	5,045	5,126	235	55,225
16	5,010	4,900	4,955	4,927	145	21,025
17	4,650	4,500	4,575	4,858	-235	55,225
18	4,720	4,910	4,815	4,782	5	25
19	5,010	5,150	5,080	4,823	270	72,900
20	4,350	4,390	4,370	4,755	-440	193,600
21	5,300	5,310	5,305	4,918	495	245,025
22	4,100	3,815	4,200	4,625	-610	372,100
23	5,600	5,575	5,588	5,031	778	604,506
24	4,750	4,890	4,820	4,869	10	100
25	4,910	4,880	4,895	5,101	85	7,225
26	4,890	5,080	4,985	4,900	175	30,625
27	4,910	5,000	4,955	4,945	145	21,025
28	4,150	3,700	3,925	4,622	-885	783,225
29	4,590	4,670	4,630	4,503	-180	32,400
30	3,990	4,550	4,270	4,275	-540	291,600
		TOTAL	**144,733**			**4,196,294**
		AVERAGE, $\bar{X}$	**4,824**			

You could use the mix design associated with this data on another project with similar specifications (4,000 psi concrete). The data would act as your evidence that the mix is adequate.

Entering concrete strength records regularly with a statistical calculator will quickly and easily establish and monitor your standard deviation. This will give you an ongoing evaluation of your batch plant quality control and concrete strength consistency. It will enable you to spot strength trends and changes.

Standard deviation gives you a sound basis for setting or changing target f'_{cr} and for proving the capability and appropriateness of your designs for other projects. In addition to being a part of good quality procedures, fine-tuning your procedures based on information provided by this statistical tool will help reduce or eliminate rejections due to low strengths, thus improving your bottom line. MC

Tables are provided courtesy of the American Concrete Institute from ACI 318-02, section 5.3.1.1, Building Code Requirements for Structural Concrete.